Psychological Game Analysis for Crowdsourcing with Reciprocity

Kun Xin[†‡], Sizheng Fan[†‡], Wei Cai[†‡*]

[†]*School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen, China*
[‡]*Shenzhen Institute of Artificial Intelligence and Robotics for Society, China*
{kunxin, sizhengfan}@link.cuhk.edu.cn, caiwei@cuhk.edu.cn

Abstract—**Incentive mechanism design in crowdsourcing is a trending topic. Advanced research attempts to tackle this issue from a game-theory perspective, modeling workers' and requestor's material utility functions. Besides material benefits, studies have shown that the intrinsic rewards (psychological factors) were also part of the workers' non-negligible motivation. However, previous works only mention this discovery textually, rather than quantifying their models' psychological factors. To fill this blank, we utilize the psychological game theory to analyze the crowdsourcing process. With mathematical ways, we first show that the requestor could reduce the cost of compensation via psychology methods, substituting partial monetary rewards with psychological payoffs. Furthermore, when workers are reciprocal and risk-neutral about expected earnings, we prove that the related incentive plan is requestor's optimal choice. In particular, we find the workers' psychological payoffs and requestor's cost in equilibrium. Finally, we conduct a simulation to illustrate our findings intuitively. Therefore, our unique psychological crowdsourcing model provides a promising detour for incentive mechanism design in crowdsourcing scenarios.**

Index Terms—**Crowdsourcing, Psychological Game Theory, Reciprocity, Incentive Mechanism Design**

I. INTRODUCTION

With the prosperity of sharing economy, crowdsourcing arises as a new exemplar of cooperative work, utilizing a crowd of participants worldwide to complete a complicated task [1]. In a crowdsourcing scenario, the requestor provides remuneration to attract workers to join in. Hence, it is crucial to design an incentive mechanism to ensure workers' participation [2].

Among the techniques addressing the incentive mechanism design in crowdsourcing, game theory is frequently used to model the behaviors of the requestor and workers to maximize their utility [3]. For instance, Hu *et al.* [4] proposed two algorithms to tackle the sequential crowdsourcing dilemma problem by utilizing a zero-determinant strategy; Naroditskiy *et al.* [5] introduced the prisoner's dilemma game into modeling workers' malicious behaviors. Nevertheless, none of the previous works took *psychological factors* into account quantitatively.

Though authors in [6] have mentioned that psychological factors could serve as intrinsic rewards to attract participants into crowdsourcing, there still lacks processes to make a quantitative analysis. To fill this gap, we first apply the psychological game theory to crowdsourcing game model

design for analysis. As part of the burgeoning area of behavioral economics, psychological game theory, which combines classical game theory and psychology concepts, allows the modeling of belief-dependent psychological utilities into games [7]. Specifically, several social psychologists have a common opinion that human relationships such as friendship, cooperation, and competition, correlated to the psychological conception - *reciprocity* [8], which is an opportune concept to analog the relationship between workers in crowdsourcing. The central concept of reciprocity is both appealing and straightforward: People reward those they believe behaved nicely to them, known as reciprocity. Moreover, recent work illustrates that reciprocity truly exists in the crowdsourcing system [9], especially with the support of blockchain [10].
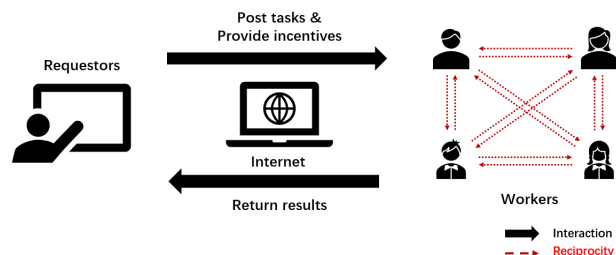

Fig. 1. Crowdsourcing with reciprocity

As Fig. 1 shows, in a crowdsourcing system: the requestor posts task and provides incentives through the Internet, workers finish the task and return results to the requestor. Among the workers, there exists reciprocity which we will focus on in this paper [9]. With psychological game theory, we express the kindness between workers and obtain their psychological utility functions. In addition, we formulate and minimize the requestor's cost function under the premise that it ensures workers' benefits. By solving the optimization problem, we obtain the equilibrium point of our model. We also compare the results under three different incentive plans: individual incentive plan, joint incentive plan, and relative incentive plan, which will be elaborated in detail in Section IV. Then the fact that psychological factors could replace partial monetary incentives got proved quantitatively. Finally, we make a simulation to illustrate our findings intuitively.

Conclusively, our contributions are summarized as follows:

- To the best of our knowledge, this is the first work utilizing psychological game theory to analyze the incentive mechanism design problem in crowdsourcing. A psychological game was formulated based on the psychological game theory, and the equilibrium was proven.
- Comparing three different incentive plans, we calculated the equilibrium psychological payoff under each plan and proved that the relative incentive plan would be the optimal choice for the requestor.
- We applied mathematical justification to prove that psychological factors could partially replace monetary incentives when introducing reciprocity to crowdsourcing systems.

The remainder of the paper is organized as follows: We first survey the related work in Section II before presenting our crowdsourcing game model in Section III. Afterward, we analyze three different incentive plans based on our model in Section IV. Section V concludes this paper.

II. RELATED WORK

With the transparent feature enabled by the blockchain technology, Xin *et al.* [10] proposed a reciprocal crowdsourcing system in which the workers can benefit from other workers and even the task itself. The experiment validated the concept of reciprocity: workers would respond bitterly to others' sinister behaviors as destroyers and respond kindly to others' friendly behaviors as guardians. In other words, workers are not merely pursuing their maximum material benefits. A worker may sacrifice part of their rewards in response to others' kindness. However, this work did not consider the phenomenon quantitatively in depth. Lu *et al.* [9] considered the reciprocity in the mechanism design to enforce cooperation and extort selfish works in crowdsourcing, but there still lacks a quantitative analysis.

III. GAME MODEL

This paper considers the scenario where a requestor launches a crowdsourcing request, then workers would follow with selective efforts. We assume that all workers are risk-neutral.

A. Game Preferences

Let $N = \{1, ..., n\}$ be the set of homogeneous workers where $n \geq 2$. Let T be the set of all the historical choice files. Let E_i be the set of all behavior strategies of worker $i \in N$. Each strategy assigns each history $t \in T$ with a probability distribution over all possible choices. Each worker i can choose effort level $e_i \in \{0, 1\}$, where $e_i = 0$ denotes that the worker selects to provide low-quality data; $e_i = 1$ denotes that the worker selects to provide high-quality data. If a worker desires to finish the job with high effort, they will pay c, representing their time and energy costs. Moreover, in providing corresponding data, worker i's effort cost can be represented as $e_i c$, measured in monetary units.

Based on the individual performance indicator $\sigma_i \in \{h, l\}$ of each worker i [11], the requestor would provide worker i

with a compensation of $p^i_{\sigma_i}$. Since each worker's task finishing process is independent, we can assume that all the indicators are uncorrelated. The probability of obtaining an indicator with high quality ($\sigma_i = h$) or low quality ($\sigma_i = l$) merely depends on worker i's individual effort: if i worked diligently, $\sigma_i = h$ with probability $\theta \in (1/2, 1)$, and if i worked idly, $\sigma_i = l$ with probability $(1 - \theta)$.

Material payoff function Worker i's material payoff function can be represented as:

$$\pi_i = p^i_{\sigma_i} - e_i c \tag{1}$$

Psychological payoff function Since we assumed all workers are reciprocal towards one another in our system, worker i's psychological payoff depends on i's beliefs on other workers' strategies and i's beliefs on other workers' beliefs on i's strategies.

In accordance with Dufwenberg and Kirchsteiger's work [12], we represent beliefs as behavior strategies. Let $A_{ij} = E_j$ denotes the set of worker i's all possible beliefs over the strategy of worker j. Let $B_{iji} = A_{ji} = E_i$ denotes the set of worker i's all possible beliefs over worker j's belief over worker i's strategy. For beliefs $e_{ij} \in E_{ij}$, $a_{ij} \in A_{ij}$, and $b_{ij} \in B_{ij}$, after worker i selects strategy e_i, they believe that they select in a way (e_i) that j's material payoff would be $\pi_j \left(e_i, (a_{ij})_{j \neq i} \right)$.

Then we define the reference point of worker i's belief over worker j:

$$\pi^{ref}_{ij} \left((a_{ij})_{j \neq i} \right) = \frac{1}{2} \cdot \left[\max \left\{ \pi_j \left(e_i, (a_{ij})_{j \neq i} \right) \mid e_i \in E_i \right\} \right. \\ \left. + \min \left\{ \pi_j \left(e_i, (a_{ij})_{j \neq i} \right) \mid e_i \in E_i \right\} \right] \tag{2}$$

Worker i's kindness towards another worker $j \neq i$ can be defined as:

$$k_{ij} \left(e_i(t), (a_{ij}(t))_{j \neq i} \right) = \pi_j \left(e_i(t), (a_{ij}(t))_{j \neq i} \right) \\ - \pi^{ref}_{ij} \left((a_{ij}(t))_{j \neq i} \right) \tag{3}$$

To simplify the equation, in the rest parts, we will ignore the t. Then worker i's beliefs over how much kindness worker $j \neq i$ gives back can be expressed as:

$$\lambda_{iji} (a_{ij}, b_{iji}) = \pi_i (a_{ij}, b_{iji}) - \pi^{ref}_{ij} (b_{iji}) \tag{4}$$

Hence, worker i's psychological payoff function can be expressed as:

$$P_i(k_{ij}, \lambda_{iji}) = \sum_{j \in N \setminus \{i\}} \left(\varepsilon_{ij} \operatorname{sign} \left(k_{ij} (e_i, a_{ij}) \cdot \lambda_{iji} (a_{ij}, b_{iji}) \right) \right. \\ \left. \cdot \sqrt{\left| k_{ij} (e_i, a_{ij}) \cdot \lambda_{iji} (a_{ij}, b_{iji}) \right|} \right) \tag{5}$$

where the constant ε_{ij} measures the workers' inherent sensitivity concerning reciprocity. This type of representation can make the rest of the calculation process easier.

Worker i's utility function Worker i's utility is the sum of their material payoff function and psychological payoff function, which can be expressed as:

$$U_i\left(e_i, (a_{ij}, b_{iji})_{j \neq i}\right) = \pi_i\left(e_i, (a_{ij})_{j \neq i}\right) +$$
$$\sum_{j \in N \setminus \{i\}} \left(\varepsilon_{ij} \cdot \text{sign}\left(k_{ij}\left(e_i, a_{ij}\right) \cdot \lambda_{iji}\left(a_{ij}, b_{iji}\right)\right)\right. \quad (6)$$
$$\left. \cdot \sqrt{\left|k_{ij}\left(e_i, a_{ij}\right) \cdot \lambda_{iji}\left(a_{ij}, b_{iji}\right)\right|}\right)$$

Where worker i's utility is constituted with n terms, the first term is i's material payoff, and the rest is i's psychological payoffs concerning each other player $j \neq i$.

B. Problem Formulation

We denote $p_{\sigma_i \sigma_j}^i$ to be the compensation paid to worker i after both σ_i and σ_j realize. we can devise this problem into an optimization problem, in which the requestor tries to minimize the cost to motivate the workers. To simplify the presentation, we assume that there are only two workers ($i \in \{x, y\}$) in our crowdsourcing system. In the following part, we will make quantity analyses concerning the three incentive plans. By formulating the requestor's expected compensation cost, we can express the problem as a principal (requestor) problem:

$$\min \quad \begin{aligned} &\theta^2 \sum_{i \in x,y} p_{hh}^i + (1-\theta)\theta \sum_{i \in x,y} \left(p_{hl}^i + p_{lh}^i\right) \\ &+ (1-\theta)^2 \sum_{i \in x,y} p_{ll}^i \end{aligned}$$

$$\text{s.t.} \quad U_i(1,1,1) \geq U_i(0,1,1),$$
$$\forall i \in x, y$$
$$p_{hh}^i, p_{hl}^i, p_{lh}^i, p_{ll}^i \geq 0$$

where $U_i(1,1,1) \geq U_i(0,1,1)$ is the incentive compatibility constraints (IC). IC is used to ensure that when we obtain the sequential reciprocity equilibrium [12], worker i's effort level in the optimal incentive plan can reach the high effort $\{e_i = 1 \mid i \in x, y\}$. High effort level is usually preferred by the requestor, so we only concentrate on the pure strategy equilibrium where $\{e_i = 1 \mid i \in x, y\}$ in this paper. Worker $i \in \{x, y\}$'s utility function could be rewritten as:

$$U_i\left(e_i, a_{ij}, b_{iji}\right) = \pi_i\left(e_i, a_{ij}\right) +$$
$$\varepsilon_{ij} \cdot \text{sign}\left(k_{ij}\left(e_i, a_{ij}\right) \cdot \lambda_{iji}\left(a_{ij}, b_{iji}\right)\right)$$
$$\cdot \sqrt{\left|k_{ij}\left(e_i, a_{ij}\right) \cdot \lambda_{iji}\left(a_{ij}, b_{iji}\right)\right|} \quad (8)$$

Combine Eq.(7) and Eq.(8), the incentive compatibility constraints (IC) could be represented as:

$$\theta^2 p_{hh}^i + (1-\theta)\theta(p_{hl}^i + p_{lh}^i) + (1-\theta)^2 p_{ll}^i - 1 * c +$$
$$\varepsilon_{ij} \cdot \text{sign}\left(k_{ij}\left(1,1\right) \cdot \lambda_{iji}\left(1,1\right)\right) \cdot \sqrt{\left|k_{ij}\left(1,1\right) \cdot \lambda_{iji}\left(1,1\right)\right|} \geq$$
$$(1-\theta)\theta p_{hh}^i + (1-\theta)^2 p_{hl}^i + \theta^2 p_{lh}^i + (1-\theta)\theta p_{ll}^i - 0 * c +$$
$$\varepsilon_{ij} \cdot \text{sign}\left(k_{ij}\left(0,1\right) \cdot \lambda_{iji}\left(1,1\right)\right) \cdot \sqrt{\left|k_{ij}\left(0,1\right) \cdot \lambda_{iji}\left(1,1\right)\right|} \quad (9)$$

IV. RESULT ANALYSIS

The requestor usually has three incentive plans to implement [13]: (1) The individual incentive plan (IIP) rewards the worker only based on their performance, unrelated to peer performance indicator; (2) The relative incentive plan (RIP) rewards the worker while the quality of their result is superior to others; (3) The joint incentive plan (JIP) rewards the worker under the condition that their peer performs well.

A. No Reciprocity

We assume there exists no reciprocity among workers in the first place, neglecting the workers' psychological effects ($\varepsilon = 0$). Then the incentive compatibility constraints (IC) is as follows:

$$\theta^2 p_{hh}^i + (1-\theta)\theta(p_{hl}^i + p_{lh}^i) + (1-\theta)^2 p_{ll}^i - c \geq$$
$$(1-\theta)\theta p_{hh}^i + (1-\theta)^2 p_{hl}^i + \theta^2 p_{lh}^i + (1-\theta)\theta p_{ll}^i \quad (10)$$

For individual incentive plan (IIP), the worker gets rewards when they provide result with high quality (choosing high effort level), indicating $p_{lh}^i = p_{ll}^i = 0$. Substitute that into above, we obtain $p_{hh}^i = p_{hl}^i = \frac{c}{(2\theta-1)}$; For the relative incentive plan (RIP), the worker gets rewards when they put in higher effort comparing to others, indicating $p_{lh}^i = p_{ll}^i = p_{hh}^i = 0$. Similarly, we obtain $p_{hl}^i = \frac{c}{(2\theta-1)(1-\theta)}$; For the joint incentive plan (JIP), out of convenience, we consider the following particular case: the worker gets rewards only if both they and their peer perform well, indicating $p_{hl}^i = p_{lh}^i = p_{ll}^i = 0$. Similarly, we obtain $p_{hh}^i = \frac{c}{(2\theta-1)\theta}$

B. With Reciprocity

In this part, we consider the case that there exists reciprocity among workers.

Theorem 1 When the game reaches equilibrium, the effort level of x and y is $\{e_x = 1, e_y = 1\}$ for all incentive plans IIP, RIP and JIP.

Proof: To prove the condition that both workers utilize positive efforts as a psychological equilibrium, the overall utility of choosing $e_i = 1$ must be greater than that of choosing $e_i = 0$ given $a_{ij} = b_{iji} = 1 \; \forall i, j \in \{x, y\}$. Then we can check each incentive plan individually.

When the requestor chooses IIP: there is no reciprocity among workers, so worker i's compensation is independent to σ_j and e_j. Therefore, when both workers choose the high effort level, the game's equilibrium is reached.

When the requestor chooses JIP: given $a_{ij} = 1$, indicating i thinks that j would choose to provide high quality data. Worker i supposes that status $\{\sigma_i = h, \sigma_j = h\}$ would reach with probability θ^2 that i works diligently, and with probability $\theta(1-\theta)$ that i works idly. In JIP, i's high effort is a sign of kindness to j as it enlarges j's expected material payoff. Thus, after i has a first order belief of $a_{ij} = 1$, i's expectant kindness towards j choosing effort level $e_i = 1$ is:

$$k_{ij}(1,1) = \theta^2 p_{hh}^j - c - \frac{1}{2}\left(\theta^2 p_{hh}^j - c + \theta(1-\theta)p_{hh}^j - c\right) \quad (11)$$

where the parenthesis term represents j's reference point payoff given i's beliefs. Substitute the corresponding $p^i_{\sigma_i \sigma_j}$ from section IV-A into the function, the expression above can be rewritten as:

$$k_{ij}(1,1) = \frac{\theta(2\theta-1)p^j_{hh}}{2} = \frac{c}{2} \qquad (12)$$

which always remains positive since i acting kindly towards j made it easier for j to get rewards. If i decides to loaf on the job, i's expected kindness to j is:

$$k_{ij}(0,1) = -\frac{c}{2} \qquad (13)$$

which always remains negative since comparing to j's equitable payoff, i's expected payoff given to j is lower. Similarly, i's expected received kindness from j is given by:

$$\lambda_{iji}(1,1) = \frac{\theta(2\theta-1)p^i_{hh}}{2} = \frac{c}{2} \qquad (14)$$

Therefore, when i selects the high effort level, they expect a positive psychological payoff; when i selects the low effort level, they expect a negative psychological payoff.

Hence, when the requestor chooses JIP and $a_{ij} = b_{iji} = 1$, worker i's utility can be expressed as:

$$\frac{\theta^2 c}{(2\theta-1)\theta} - c + \varepsilon\frac{c}{2} \qquad (15)$$

when i choose $e_i = 1$, the utility will be:

$$\frac{\theta(1-\theta)c}{(2\theta-1)\theta} - \varepsilon\frac{c}{2} \qquad (16)$$

It is evident that Eq.(15) is (strictly) greater than Eq.(16) for any $\varepsilon \geq (>)0$.

When the requestor chooses RIP: given $a_{ij} = b_{iji} = 1$ $\forall i, j \in \{x,y\}$, take the correspond $p^i_{\sigma_i \sigma_j}$ from section IV-A and substitute it into the function. When selecting $e_i = 1$, worker i's expected kindness towards worker j is:

$$k_{ij}(1,1) = \theta(1-\theta)p^j_{hl} - c - \frac{1}{2}\left[\theta(1-\theta)p^j_{hl} - 2c + \theta^2 p^j_{hl}\right]$$
$$= -\frac{\theta(2\theta-1)p^j_{hl}}{2} = -\frac{\theta c}{2(1-\theta)} \qquad (17)$$

which remains negative. j's expected payoff decreases as i chooses high effort. Conversely, when worker i chooses $e_i = 0$, i's expected kindness towards worker j is represented as:

$$k_{ij}(0,1) = \frac{\theta c}{2(1-\theta)} \qquad (18)$$

which remains positive. Similarly, i's expected received kindness from j is represented as:

$$\lambda_{iji}(1,1) = -\frac{\theta(2\theta-1)p^i_{hl}}{2} = -\frac{\theta c}{2(1-\theta)} \qquad (19)$$

Thus, worker i's total utility is represented as:

$$(1-\theta)\theta\frac{c}{(2\theta-1)(1-\theta)} + \varepsilon\frac{\theta c}{2(1-\theta)} - c \qquad (20)$$

when i chooses $e_i = 1$, the utility will be:

$$(1-\theta)^2\frac{c}{(2\theta-1)(1-\theta)} - \varepsilon\frac{\theta c}{2(1-\theta)} \qquad (21)$$

Same as above, it's evident that Eq.(20) is (strictly) greater than Eq.(21) for any $\varepsilon \geq (>)0$.

Theorem 2 Under different incentive plans, worker i's expected received equilibrium psychological payoff at stage 2 is:

$$P_i(k_{ij}, \lambda_{iji}) = \begin{cases} 0 & \text{under (IIP)} \\ \frac{\varepsilon c}{2} & \text{under (JIP)} \\ \frac{\varepsilon \theta c}{2(1-\theta)} & \text{under (RIP)} \end{cases}$$

Proof: According to the first part of the proof in Theorem 1, IIP's entailed psychological payoff equals zero.

JIP's entailed psychological payoff equals to $\frac{\varepsilon c}{2}$ is verified from Eq.(15). The overall psychological incentive derives from εc and could verify from Eq.(15) and Eq.(16). Similarly, RIP's entailed psychological payoff equal to $\frac{\varepsilon \theta c}{2(1-\theta)}$ is verified from Eq.(20). The overall psychological incentive derives from $\frac{\varepsilon \theta c}{(1-\theta)}$ and could verify from Eq.(20) and Eq.(21).

Theorem 3 When we introduce reciprocity to the crowdsourcing system and the workers are risk-neutral about the amount of compensation, we can obtain three local solution functions for the requestor problem: 1. The normal individual incentive plan, which is the same to Section IV-A; 2. joint incentive plan in which:

$$p^i_{hh} = \frac{c}{\theta(2\theta-1)(1+\varepsilon)} > p^i_{hl} = p^i_{lh} = p^i_{ll} = 0 \qquad (22)$$

3. and relative incentive plan in which:

$$p^i_{hl} = \frac{c}{(2\theta-1)(1-\theta+\theta\varepsilon)} > p^i_{ll} = p^i_{hh} = p^i_{lh} = 0 \qquad (23)$$

for all $i \in \{x,y\}$.

Proof: For the principal (requestor)'s problem 7, with $i, j \in \{x,y\}$, we have:

$$k_{ij}(1,1) = \frac{(2\theta-1)}{2}\left[\theta\left(p^j_{hh}-p^j_{hl}\right) + (1-\theta)\left(p^j_{lh}-p^j_{ll}\right)\right]$$
$$= \lambda_{jij}(1,1) \qquad (24)$$

$$k_{ij}(0,1) = -\frac{(2\theta-1)}{2}\left[\theta\left(p^j_{hh}-p^j_{hl}\right) + (1-\theta)\left(p^j_{lh}-p^j_{ll}\right)\right] \qquad (25)$$

Then we can formulate the Lagrangian of the requestor problem:

$$L = \theta^2 \sum_{i \in \{x,y\}} p^i_{hh} + (1-\theta)\theta \sum_{i \in \{x,y\}} \left(p^i_{hl} + p^i_{lh}\right)$$
$$+ (1-\theta)^2 \sum_{i \in \{x,y\}} p^i_{ll}$$
$$- \mu_x\left[(2\theta-1)*(\theta\left(p^x_{hh}-p^x_{lh}\right)+(1-\theta)p^x_{hl} - p^x_{ll}\right)$$
$$-c + \varepsilon P_x\left(\lambda_{xyx}(11), k_{xy}(11)\right) - P_x\left(\lambda_{xyx}(11), k_{xy}(01)\right)]$$
$$- \mu_y\left[(2\theta-1)*(\theta\left(p^y_{hh}-p^y_{lh}\right)+(1-\theta)p^y_{hl} - p^y_{ll}\right)$$
$$-c + \varepsilon P_y\left(\lambda_{yxy}(11), k_{yx}(11)\right) - P_y\left(\lambda_{yxy}(11), k_{yx}(10)\right)] \qquad (26)$$

By taking the derivation of Lagrangian, we can obtain:

$$
\begin{aligned}
\frac{\partial L}{\partial p_{hh}^i} =&\theta^2 - \mu_i\theta(2\theta-1)(p_{hh}^i)' + \\
&\mu_i\varepsilon\left[\frac{\partial P_i\left(\lambda_{iji}(1,1),k_{ij}(1,1)\right)}{\partial p_{hh}^i} - \right. \\
&\left.\frac{\partial P_i\left(\lambda_{iji}(1,1),k_{ij}(0,1)\right)}{\partial p_{hh}^i}\right] - \\
&\mu_j\varepsilon\left[\frac{\partial P_j\left(\lambda_{jij}(1,1),k_{ji}(1,1)\right)}{\partial p_{hh}^i} - \right. \\
&\left.\frac{\partial P_j\left(\lambda_{jij}(1,1),k_{ji}(0,1)\right)}{\partial p_{hh}^i}\right] \geq 0
\end{aligned}
\tag{27}
$$

Similarly, we can obtain $\frac{\partial L}{\partial p_{hl}^i}, \frac{\partial L}{\partial p_{lh}^i}, \frac{\partial L}{\partial p_{ll}^i}$.

If there exists no reciprocity in the crowdsourcing system, which means $\lambda_{iji}(1,1) = k_{ij}(1,1) = k_{ij}(0,1) = 0$, then by solving the derivation of Lagrangian we obtain the same solution as the individual incentive plan, in which the worker's reward function does not contain any psychological utility.

If there exists positive reciprocity in the crowdsourcing system, which means $\lambda_{iji}(1,1) = k_{ij}(1,1) > 0 > k_{ij}(0,1)$, and given that all workers are identical, then we can rewrite the derivation of Lagrangian as:

$$
\frac{\partial L}{\partial p_{hh}^i} \geq 0 \Rightarrow \theta^2 - \mu[\theta(2\theta-1)(1+\varepsilon)](p_{hh}^i)' \geq 0
\tag{28}
$$

$$
\frac{\partial L}{\partial p_{hl}^i} \geq 0 \Rightarrow \theta(1-\theta) - \mu[(2\theta-1)(1-\theta-\varepsilon\theta)](p_{hl}^i)' \geq 0
\tag{29}
$$

$$
\frac{\partial L}{\partial p_{lh}^i} \geq 0 \Rightarrow (1-\theta)\theta - \mu[(2\theta-1)(-\theta+\varepsilon(1-\theta))](p_{lh}^i)' \geq 0
\tag{30}
$$

$$
\frac{\partial L}{\partial p_{ll}^i} \geq 0 \Rightarrow (1-\theta)^2 + \mu[(2\theta-1)(1+\varepsilon)(1-\theta)](p_{ll}^i)' \geq 0
\tag{31}
$$

It is evident that $\mu > 0$. From conditions Eq.(28)-Eq.(31) we can see that $p_{lh}^i = p_{ll}^i = 0$ if $\varepsilon \in [0, \frac{1-\theta}{\theta}]$ and $p_{hl}^i = p_{lh}^i = p_{ll}^i = 0$ if $\varepsilon \in [\frac{1-\theta}{\theta}, 1]$. This implies that the requestor can use the joint incentives plan to generate the most valid positive psychological exchange. If $\varepsilon \in [0, \frac{1-\theta}{\theta}]$, combines with Eq.(28) and Eq.(29), we can obtain:

$$
\frac{1}{(p_{hh}^i)'} = \frac{\mu(2\theta-1)(1+\varepsilon)}{\theta}
$$

$$
\frac{1}{(p_{hl}^i)'} = \frac{\mu(2\theta-1)(1-\theta-\varepsilon\theta)}{\theta(1-\theta)}
$$

that leads to:

$$
p_{hl}^i = \frac{(1-\theta-\varepsilon\theta)}{(1+\varepsilon)(1-\theta)}p_{hh}^i
\tag{32}
$$

If $\varepsilon \in [\frac{1-\theta}{\theta}, \frac{\theta}{1-\theta}]$, from above we can know that p_{hh}^i remains positive. Substituting this into Eq.(9):

$$
(2\theta-1)(1+\varepsilon)\theta p_{hh}^i = c
\tag{33}
$$

which leads to:

$$
p_{hh}^i = \frac{c}{(2\theta-1)(1+\varepsilon)\theta}
\tag{34}
$$

Here, we prove the Eq.(22).

If there exists negative reciprocity in the crowdsourcing system, which means $\lambda_{iji}(1,1) = k_{ij}(1,1) < 0 < k_{ij}(0,1)$, and given that all workers are identical, then we can rewrite the derivation of Lagrangian as:

$$
\frac{\partial L}{\partial p_{hh}^i} \geq 0 \Rightarrow \theta^2 - \mu[\theta(2\theta-1)(1-\varepsilon)](p_{hh}^i)' \geq 0
\tag{35}
$$

$$
\frac{\partial L}{\partial p_{hl}^i} \geq 0 \Rightarrow \theta(1-\theta) - \mu[(2\theta-1)(1-\theta+\varepsilon\theta)](p_{hl}^i)' \geq 0
\tag{36}
$$

$$
\frac{\partial L}{\partial p_{lh}^i} \geq 0 \Rightarrow (1-\theta)\theta + \mu[(2\theta-1)(\theta+\varepsilon(1-\theta))](p_{lh}^i)' \geq 0
\tag{37}
$$

$$
\frac{\partial L}{\partial p_{ll}^i} \geq 0 \Rightarrow (1-\theta)^2 + \mu[(2\theta-1)(1-\varepsilon)(1-\theta)](p_{ll}^i)' \geq 0
\tag{38}
$$

It is evident $\mu > 0$. From conditions Eq.(35)-Eq.(38) we can see that $p_{lh}^i = p_{ll}^i = 0$ if $\varepsilon \in [0, 1]$ and $p_{hh}^i = p_{lh}^i = p_{ll}^i = 0$ if $\varepsilon = 1$. We can know that the requestor can use the relative incentives plan to generate the most valid negative psychological exchange. If $\varepsilon \in [0, 1]$, combines with Eq.(35) and Eq.(36), we can obtain:

$$
\frac{1}{(p_{hh}^i)'} = \frac{\mu(2\theta-1)(1-\varepsilon)}{\theta}
$$

$$
\frac{1}{(p_{hl}^i)'} = \frac{\mu(2\theta-1)(1-\theta+\varepsilon\theta)}{\theta(1-\theta)}
$$

that leads to:

$$
p_{hh}^i = \frac{(1-\varepsilon)(1-\theta)}{(1-\theta+\theta\varepsilon)}p_{hl}^i
\tag{39}
$$

If $\varepsilon = 1$ from above we know that only p_{hl}^i remains positive. Substituting this into the Eq.(9):

$$
(2\theta-1)(1-\theta+\theta\varepsilon)p_{hl}^i = c
\tag{40}
$$

which leads to:

$$
p_{hl}^i = \frac{c}{(2\theta-1)(1-\theta+\theta\varepsilon)}
\tag{41}
$$

Here we proved Eq.(23).

Theorem 4 When we introduce reciprocity to the crowdsourcing system and the workers are risk-neutral about the amount of compensation, the optimal option for the requestor is to provide incentives through the relative incentive plan.

Proof: From the Theorem 3, we can know that the expected cost of the requestor is:

$$
E(k_{ij}, \lambda_{iji}) = \begin{cases} \frac{2\theta c}{(2\theta-1)} & \text{IIP} \\ \frac{2\theta^2 c}{(2\theta-1)(1+\varepsilon)\theta} & \text{JIP} \\ \frac{2\theta(1-\theta)c}{(2\theta-1)(1-\theta+\theta\varepsilon)} & \text{RIP} \end{cases}
$$

Simple algebra shows that the expected cost under RIP is strictly smaller than JIP and IIP for any $\varepsilon > 0$. Moreover, compared with results of Theorem 3 with Section IV-A, it also proves that we can replace partial monetary rewards with psychological incentives by introducing reciprocity to the system design.
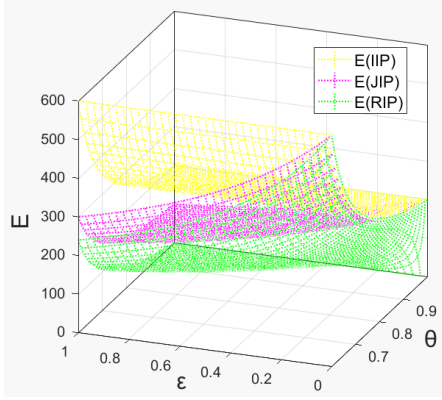
C. Simulation

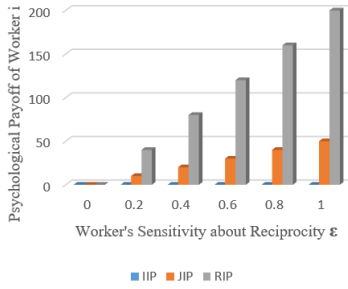Fig. 2. Expected cost of requestor when $c = 100$

Fig. 3. Psychological payoff of worker when $c = 100$ and $\theta = 0.8$

To illustrate the theorem more intuitively, we simulate the case when $c = 100$. As shown in Fig. 2, the x-coordinate and y-coordinate represent the ε and θ respectively, the z-coordinate represents the expected cost of the requestor. Evidently, the green surface is strictly below the yellow and pink surface when $\varepsilon \neq 0$, which means that the requestor's expected cost is lower under RIP than JIP and IIP when there exists reciprocity. In other words, RIP provides more significant psychological incentives than JIP. Besides, we can see that their faces curve down for pink and green surfaces as the ε increases. Specifically, the line when $\varepsilon = 0$ is the highest line of that surface shows that we can reduce the requestor's cost by introducing reciprocity. If we let $\theta = 0.8$ as Fig. 3 shows, the psychological payoff of worker at equilibrium increases when the worker becomes more sensitive to reciprocity. And the relative incentive plan (RIP) can provide the highest psychological payoff to worker, which is consistent with our theorem.

V. Conclusion

In this article, we examine crowdsourcing games from both typical game theory perspectives and psychological effects. Notably, we introduce reciprocity into the crowdsourcing system and make a quantitative analysis. Under three different incentive plans, we find their equilibrium points, respectively. Furthermore, we prove that the relative incentive mechanism is the optimal choice for the requestor when the workers are reciprocal and risk-neutral concerning wealth variations. One of the most significant contributions of this paper is that we mathematically prove that psychological incentives could replace monetary rewards partially. To our best knowledge, our proposed crowdsourcing game model is precursory and elementary. Moreover, we believe the analysis in this work will benefit reciprocity in crowdsourcing supported by blockchain, which are transparent and auditable.

VI. Acknowledge

References

[1] C. L. Tucci, A. Afuah, and G. Viscusi, *Creating and capturing value through crowdsourcing*. Oxford University Press, 2018.

[2] Y. Zhang, C. Jiang, L. Song, M. Pan, Z. Dawy, and Z. Han, "Incentive mechanism for mobile crowdsourcing using an optimized tournament model," *IEEE Journal on Selected Areas in Communications*, vol. 35, no. 4, pp. 880–892, 2017.

[3] L. Xiao, T. Chen, C. Xie, H. Dai, and H. V. Poor, "Mobile crowd-sensing games in vehicular networks," *IEEE Transactions on Vehicular Technology*, vol. 67, no. 2, pp. 1535–1545, 2017.

[4] Q. Hu, S. Wang, P. Ma, X. Cheng, W. Lv, and R. Bie, "Quality control in crowdsourcing using sequential zero-determinant strategies," *IEEE Transactions on Knowledge and Data Engineering*, vol. 32, no. 5, pp. 998–1009, 2019.

[5] V. Naroditskiy, N. R. Jennings, P. Van Hentenryck, and M. Cebrian, "Crowdsourcing dilemma," *arXiv preprint arXiv:1304.3548*, 2013.

[6] J. Hamari, D. J. Shernoff, E. Rowe, B. Coller, J. Asbell-Clarke, and T. Edwards, "Challenging games help students learn: An empirical study on engagement, flow and immersion in game-based learning," *Computers in human behavior*, vol. 54, pp. 170–179, 2016.

[7] O. H. Azar, "The influence of psychological game theory," *Journal of Economic Behavior & Organization*, vol. 167, pp. 445–453, 2019.

[8] D. A. Bosse, R. A. Phillips, and J. S. Harrison, "Stakeholders, reciprocity, and firm performance," *Strategic Management Journal*, vol. 30, no. 4, pp. 447–456, 2009.

[9] J. Lu, Y. Xin, Z. Zhang, S. Tang, C. Tang, and S. Wan, "Extortion and cooperation in rating protocol design for competitive crowdsourcing," *IEEE Transactions on Computational Social Systems*, vol. 8, no. 1, pp. 246–259, 2020.

[10] K. Xin, S. Zhang, X. Wu, and W. Cai, "Reciprocal crowdsourcing: Building cooperative game worlds on blockchain," in *2020 IEEE International Conference on Consumer Electronics (ICCE)*, pp. 1–6, IEEE, 2020.

[11] B. Holmstrom, "Moral hazard in teams," *The Bell Journal of Economics*, pp. 324–340, 1982.

[12] M. Dufwenberg and G. Kirchsteiger, "A theory of sequential reciprocity," *Games and economic behavior*, vol. 47, no. 2, pp. 268–298, 2004.

[13] M. London and G. R. Oldham, "A comparison of group and individual incentive plans," *Academy of Management Journal*, vol. 20, no. 1, pp. 34–41, 1977.